# Activity #1: Understanding Sinusoids Worksheet
# (Teacher version)
# Math

**Note to students: Lab teams of three or four students are required for this activity.**

**National Standards addressed:**

**Content Standards:**
**Algebra Expectations: Students will understand and perform transformations such as combining, composing, and inverting commonly used functions, using technology to perform such operations on more complicated symbolic expressions; students will use symbolic algebra to represent and explain mathematical relationships; students will judge the meaning, utility and reasonableness of the results of symbolic manipulations, including those carried out by technology; students will identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships; students will draw reasonable conclusions about the situation being modeled.**

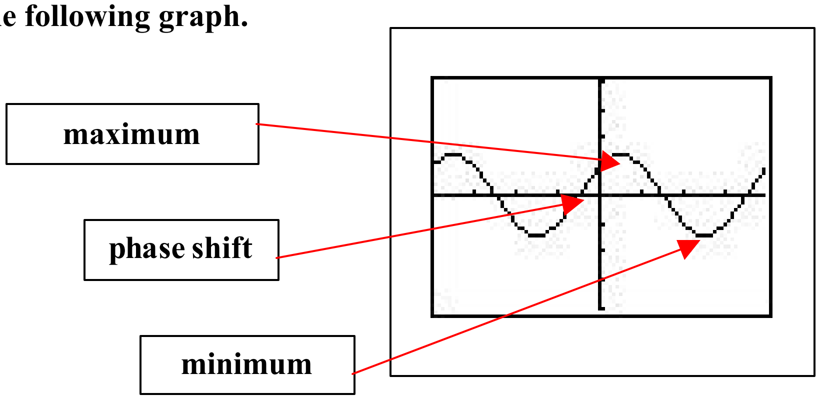
**Process Standards:**
**Measurement Expectation: Students will make decisions about units and scales that are appropriate for problem situations involving measurement.**
**Problem Solving Expectation: Students will reflect on the process of mathematical problem solving.**
**Communication Expectation: Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.**
**Connection Expectation: Students will recognize and apply mathematics in contexts outside of mathematics.**
**Representation Expectation: Students will use representations to model and interpret physical, social, and mathematical phenomena.**

**Purpose:**
- **To understand the definition of sinusoidal curve**
- **To recognize a possible sinusoidal curve**
- **To express a curve in sinusoidal form**
- **To generate, using technology, a sine curve and find its mathematical representation**
- **To recognize phenomena that might generate these curves and discuss advantages of the sine representation**

# Understanding Sinusoids → y = a sin (bx - c) + d

**Definition: A sinusoid is the name given to any curve that can be written in the form, y = a sin (bx - c) + d.**

**To investigate, you will look at several curves, determine whether or not these curves are representative of the sine curve, and, if so, rewite in sine form. Consider the curve with equation y = sin x + cos x. Using radian mode, the curve has the following graph.**




**This graph does look like a sine curve. You will need to identify some points and do some calculations before you can make your sinusoidal representation (if possible)**

**.**

**1. Identify the coordinates of the maximum point, $(x_1, y_1)$. Give coordinates correct to three decimal places.**

**$(x_1 =$ ___________, $y_1 =$ ___________) For best results, store this x-value in A and this y-value in B.** **($x_1 = .785$, $y_1 = 1.414$) Hopefully, students will recognize these two values as $\frac{p}{4}$ and $\frac{p}{2}\sqrt{2}$, respectively.**

**2. Identify the coordinates of the minimum point, $(x_2, y_2)$. Give coordinates correct to three decimal places.**

**$(x_2 =$ ___________, $y_2 =$ ___________)**

**For best results, store this x-value in C and this y-value in D.**

**($x_2 = 3.927$, $y_2 = -1.414$) Hopefully, students will recognize these two values as $\frac{5p}{4}$ and $-\sqrt{2}$, respectively.**

**3. Identify the coordinates of the phase shift point, $(x_3, y_3)$. Give coordinates correct to three decimal places.**

**$(x_3 =$ ___________, $y_3 =$ ___________)**

**For best results, store this x-value in E and this y-value in F.**

**($x_3 = -.785$, $y_3 = 0$) Students should easily recognize $x_3$ as $-\frac{p}{4}$.**

**You are now ready to decide on values for a, b, c, and d in the sinusoidal representation. Be sure to be as accurate as possible, i.e., use the values you have stored in your calculator.**

**4. The value of a represents the amplitude and opening direction of the curve. Write your value for a, correct to three decimal places. ______________ For best results, store this value in your calculator. What variable did you select to store a? ____**

**a = 1.414, which is the y-value stored in B.**

**5. The value for b affects the period of the graph. Determine the period from the points found in steps 1 and 2. Remember that b is a positive real number. What period did you find, correct to three decimal places? ______________ Remember, the period is the length of one complete cycle and is given by, period = $\frac{2p}{b}$.**

**For best results, store this value in your calculator. What variable did you select to store b? ____**

**period = 6.283, which is 2p. 1 is the value for b and does not need to be stored.**

**6. The value of c affects the phase shift. Remember, the phase shift is given by $\frac{c}{b}$.**

**If c is negative, the phase shift is left and if c is positive, the phase shift is right. Determine the value for c. What was your value for c, correct to three deciamal places? __________ For best results, store this value in your calculator. What variable did you select to store c? ____**

**c = -.785, which is - $\frac{p}{4}$. This value is already stored in E.**

**7. Lastly, you need to calculate the value for d. Remember that d describes the vertical shift in the graph. What value did you calculate for d, correct to three decimal places? __________**

**Since the maximum is $\sqrt{2}$ and the minimum is - $\sqrt{2}$, the graph has no vertical shift. The central axis remains the horizontal line described as y = ( $\sqrt{2}$ + ${}^{-}\sqrt{2}$ ) / 2 = 0.**

**8. You are now ready to compare the curve given by y = sin x + cos x to the sinusoidal representation you have just found. Substitute the exact values for a, b, c, and d in the sinusoidal model, y = a sin (bx - c) + d, and compare the two graphs. Write your sinusoidal equation. ________________________________________**

**Do you think you have rewritten y = sin x + cos x in sinusoidal form? __________**

**Why or why not?**

______________________________________________________________________

______________________________________________________________________

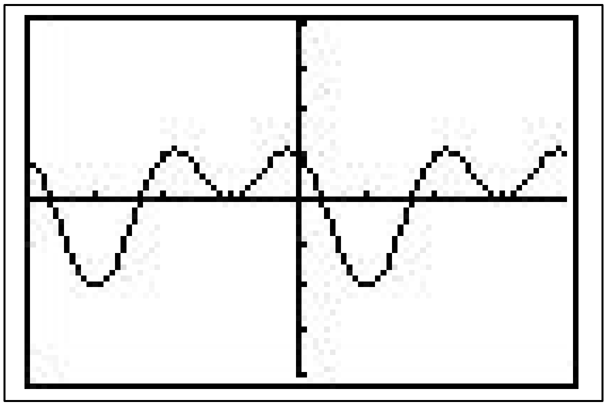
______________________________________________________________________

**y = $\sqrt{2}$ sin (1x - ${}^{-}\frac{p}{4}$) + 0**

**Determine if any of the following curves might be represented in sinusoidal form. Rewrite in sine form, those you have selected.**

**1. $y = \cos 2x - \sin x$**

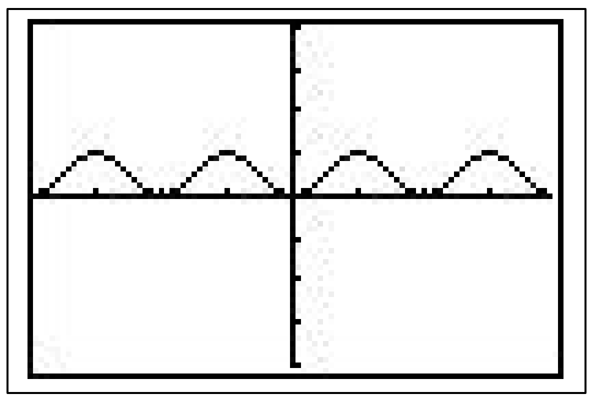
**This curve is the graph of a periodic function. However, it is not a sine curve.**



**2. $y = \sin^2 x$**

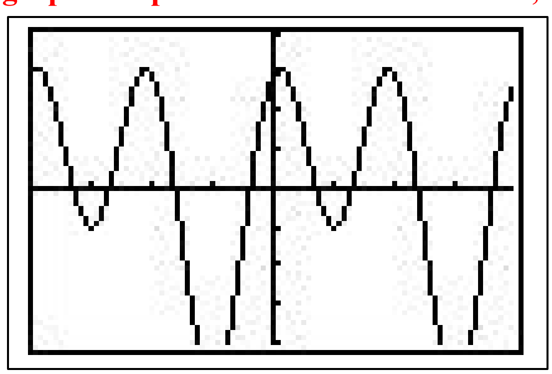
**This curve appears to be the sine curve given by the equation,**

$$y = .5 \sin\left(2x - \frac{p}{2}\right) + .5.$$



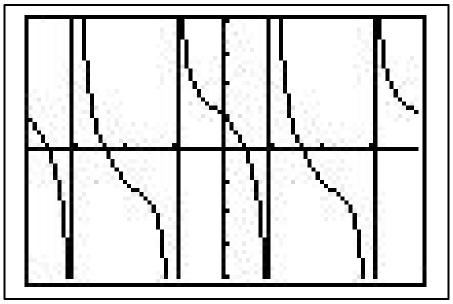
**3. $y = 3 \cos 2x + 2 \sin x$**

**This curve is the graph of a periodic function. However, it is not a sine curve.**



**4. y = cos x – tan x**

This curve is the graph of a periodic function. However, it is not a sine curve.

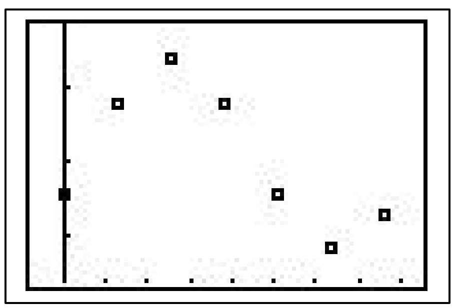


**5. Throughout the day, the depth of water at the end of a dock varies with the tides. The table shows the depths (in meters) at various times during the morning.**

| t (time) | Midnight | 2 A.M. | 4 A.M. | 6 A.M. | 8 A.M. | 10 A.M. | Noon |
|---|---|---|---|---|---|---|---|
| d (depth) | 2.55 | 3.80 | 4.40 | 3.80 | 2.55 | 1.80 | 2.27 |

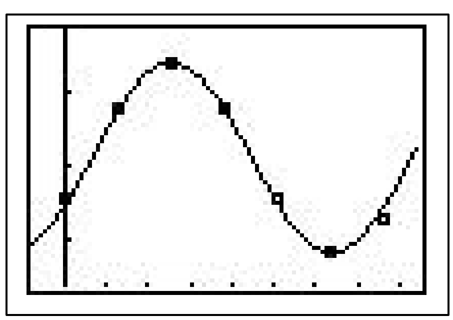
**Use a sine curve to model this data. During what times in the afternoon can a boat safely dock if it needs at least 3 meters of water to moor at the dock? (Problem from page 359 of Precalculus with Limits, by Larson, Hostetler, Edwards, and Heyd from Houghton Mifflin Ó 1997)**

Create stat plot using t-values (the x's) in list 1 and depths (the y's) in list 2.

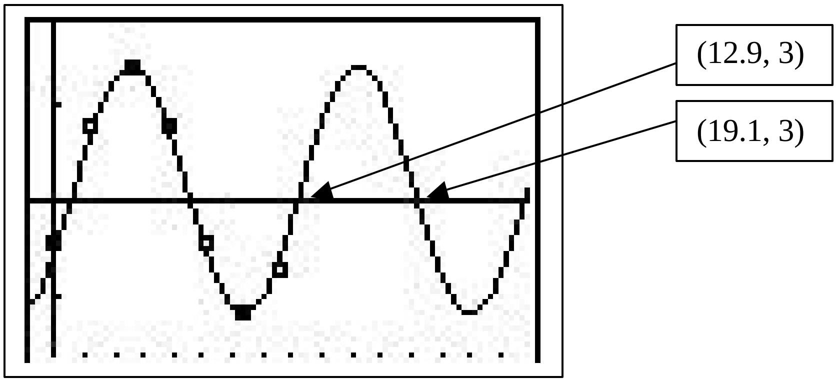


The maximum point occurs at (4, 4.4) and the minimum point occurs at (10, 1.8). From these two points, amplitude is (4.4 – 1.8)/2 = 1.3; the vertical shift is .5 (4.4 + 1.8) = 3.1. The period is 2 (10 – 4) = 12, making the value for b = $\frac{p}{6}$ and c = $\frac{p}{6}$.

This gives the equation y = $1.3\sin(\frac{p}{6}x - \frac{p}{6}) + 3.1$ $1.3\sin(\frac{p}{6}x - \frac{p}{6}) + 3.1$. Now, the two graphs appear as follows.



**Graphing the line y = 3 and changing the window so that the afternoon hours are represented by the sinusoidal curve, the depth is at least 3 meters in the afternoon between 12.9 P.M. and 19.1 P.M. or 12:51 P.M. and 7:09 P.M.**




**Link to Activity #1, Title: An Investigation into Transverse Waves, from the science component.**